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Problem solving in the mathematics curriculum: From domain-general strategies to domain-specific tactics

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Abstract

Problem solving is widely regarded as a fundamental feature within the school mathematics curriculum. However, there is considerable disagreement over what exactly problem solving is, and if and how it can be taught. In this article, I define problems as non-routine tasks and propose the explicit teaching of domain-specific problem-solving tactics that are applicable over narrow ranges of mathematical content. This is in contrast to the widespread practice of attempting to teach domain-general strategies that are supposedly applicable across diverse content areas. The proposed approach here systematically introduces students to a well-defined set of high-leverage content-specific tactics, presented in a purposeful order and taught explicitly through the use of carefully chosen problems which those tactics dramatically unlock. I argue that this sequenced approach to teaching problem-solving addresses concerns widely reported about the apparent unteachability of domain-general problem-solving strategies and has the potential to enable all students to benefit from a powerful problem-solving mathematics curriculum.

KEYWORDS

domain-general strategies, domain-specific tactics, heuristics, mathematics, Polya, problem solving

INTRODUCTION

There is a widespread consensus across many countries that problem solving is a fundamental aspect within the school mathematics curriculum, and it appears prominently in mathematics curricula around the world (Törner et al., 2007). Problem solving is critical for life in the modern world and a central aspect of mathematics (ACME, 2011, 2016; English & Gainsburg, 2016; English & Sriraman, 2010; Foster, 2019). The mathematician Paul Halmos (1980, p. 519) argued that, rather than theorems and proofs, solving problems is 'the heart of mathematics' and 'the mathematician's main reason for existence'. Today, computers can perform routine mathematical calculations far more quickly and accurately than any human. Training children in school to be merely arithmetic and algebraic human calculators, largely because these manipulations are so straightforward to assess, risks rendering the learning of mathematics an irrelevancy. Instead, the mathematics curriculum must be built around helping students to develop the creativity, ingenuity and resilience needed to approach novel problems flexibly and confidently (Foster, 2019). A small fraction of school children have always emerged from a traditional mathematics education able to do this, perhaps as a consequence of the socio-economic privileges of their background, in which they have received academic support and encouragement at home, or have somehow been lucky enough to pick up procedures quickly and discover for themselves ways to solve mathematical problems. However, leaving this to chance is clearly not an equitable approach. A large amount of research has been conducted on the teaching of problem solving in school mathematics, although there are still felt to be far more questions than answers (for reviews, see Fülöp, 2019; Olivares et al., 2021; Lesh & Zawojewski, 2007; Lester & Cai, 2016). In particular, there continues to be considerable disagreement about what exactly problem solving is and if and how it may be taught (English & Gainsburg, 2016).

For the purposes of this paper, following Christiansen and Walther (1986), I consider a mathematical *task* to be anything mathematical that a student is asked to do—whether written, oral or practical—and I divide tasks into *routine tasks* and *non-routine tasks*. Routine tasks, termed *exercises*, the mainstay of school mathematics, are invitations for students to imitate a prescribed method which they have been taught. The student should know before they begin the exercise exactly what they need to do to complete it. Developing fluency in routine tasks is extremely important for students' success in the subject and as a basis for tackling other kinds of tasks (Foster, 2018). The other category of tasks is non-routine tasks, which I term *problems*. A non-routine task, or problem, is 'a task for which the solution method is not known in advance' (NCTM, 2000, p. 52), and so there is something 'problematic' for the student when facing such a task. They do *not* know before they begin exactly how they will tackle the task, and engaging in non-routine tasks is termed *problem solving*. From a cognitive load theory perspective (Tricot & Sweller, 2014), a non-routine problem is one for which the person does not have a readily available schema in long-term memory, and 'if one has ready access to a solution schema for a mathematical task, that task is an exercise and not a problem' (Schoenfeld, 1985, p. 74). This understanding of 'problem' is normative within the mathematics education literature, but I have set it out here because it is *not* how this term is understood within psychology and cognitive science research, where 'problem solving' commonly refers to completing routine, procedural exercises. This means that much research literature which appears (on the basis of a casual literature search) to be about 'problem solving' is not about problem solving as understood in the mathematics education sense (Inglis & Foster, 2018).

It has been a complaint for many decades that problem solving is not in practice as prominent within the school mathematics curriculum as it should be (Schoenfeld, 2013). On those occasions where problem solving *is* the focus of school mathematics, this often reduces to *opportunities* for students to tackle interesting problems, but students seem rarely to be taught explicitly *how* to solve them. Where students happen to stumble on productive strategies, the teacher may draw attention to these post hoc, but this does not seem to result in the majority of students reliably learning a specific, well-defined collection of high-leverage problem-solving strategies in a systematic way (Foster, 2019). In particular, lower-attaining students are found to rarely apply their mathematics to solve novel problems (Hodgen et al., 2020). There have been numerous attempts over recent decades to actively teach problem-solving strategies, but these have largely been judged unsuccessful, and it has been claimed that commonly cited problem-solving strategies are too general for their teaching to be useful to students (Schoenfeld, 2013). Instead of attempting to 'teach problem solving', it now seems to be more typical to advocate teaching all of mathematics 'through problem solving' (e.g. see Lester & Cai, 2016). However, while this may be an effective and authentic way to learn mathematical content (Liljedahl & Cai, 2021), it is not clear that this kind of approach reliably results in students developing the knowledge and skills to be powerful problem solvers when facing *novel* problems.

In this paper, I will consider the literature concerning the teaching of problem-solving strategies in mathematics and I will propose instead the explicit teaching of domain-*specific* problem-solving *tactics* that are applicable over *narrow* ranges of mathematical content. This is in sharp distinction to attempting to teach broad, domain-general problem-solving strategies—an approach which appears to have been unsuccessful over many years. Broad domain-general strategies—such as 'be systematic', 'try specific examples' and 'draw a diagram'—might appear to have the widest utility across many mathematical problems that students are likely to face. However, as we shall see in the Problem-Solving Strategies and Tactics section, such strategies are so broad that they can be hard to teach in detail and difficult for students to know how to apply in practice (Schoenfeld, 1985, Schoenfeld, 2013). By 'domain', I mean a specific, narrow area of mathematical content, such as finding missing angles in triangles by using properties such as that the angle sum of a plane triangle is 180°. The proposed approach here focuses on determining a well-defined set of domain-specific problem-solving tactics that are then taught systematically and explicitly through the use of carefully chosen problems which those tactics dramatically unlock. Following this, by examining superficially similar problems, some of which may be solved using the same tactic, and some of which may not, students develop a deep knowledge of the boundaries of the domain within which each tactic is likely to be useful. I set out this approach in some detail below and consider what might be needed for its practical implementation. My overarching question for this paper is: *How can the explicit teaching of domain-specific tactics, rather than domain-general strategies, within the mathematics curriculum enable all students to become powerful problem solvers?* In the Routine and Non-Routine Tasks section, I clarify the distinction that I will use between 'routine' tasks (i.e. exercises) and 'non-routine' tasks (i.e. problems), as this is central to the entire argument. I present a model that indicates why teaching domain-specific tactics should be preferred over teaching domain-general strategies. In the Problem-Solving Strategies and Tactics section, I contrast strategies and tactics and use examples from the mathematical content area of angles to illustrate this important distinction. In the Explicit Teaching of Problem-Solving Tactics section, I set out an approach to how domain-specific tactics might be taught explicitly, before concluding the paper.

ROUTINE AND NON-ROUTINE TASKS

Since, as stated above, a non-routine task, or *problem*, is "a task for which the solution method is not known in advance" (NCTM, 2000, p. 52), whether any particular task constitutes a problem or not will depend on the individual and their particular prior knowledge—namely, what is available to them at that moment (Foster, 2021). For example, multiplying two 2-digit numbers together would be routine for someone well-rehearsed in using the long multiplication algorithm on such products, whereas for someone without this knowledge it would be a non-routine problem, for which they would need to try to devise an approach. In practice, when teaching a class of students who, despite variations in their prior attainment, have experienced similar prior teaching, there may often be a considerable amount of commonality in what they experience as routine or non-routine. If they have been taught a method for a particular mathematical process, then that process should in theory be routine to the entire class. However, this will of course depend on the extent to which students have retained the method that they have been taught, have competence in using it and see its relevance when presented with a task to which it could be applied.

It is important to note that the routine/non-routine categorization is distinct from low/high cognitive demand. A task may be routine but hard. For example, multiplying two 5-digit numbers together would be completely routine for someone well-acquainted with the long multiplication algorithm. However, they might nonetheless experience it as challenging, time-consuming and perhaps error-prone. A routine task may be multi-step and lengthy, but it is still routine if the person knows exactly what they need to do before they begin. The fact that they have access to an algorithm (even a demanding one) makes the task still routine.

The specific details of any problem, including how the person chooses to approach it, may also be critical in determining the routine/non-routine distinction. For example, if the two 5-digit numbers that the person was asked to multiply together happened to be 11,111 and 11,111, then they might choose to treat this as a *non-routine problem*. Instead of deploying the long multiplication algorithm that they have at their disposal, they might instead seek to exploit the repeating-digit (repdigit) nature of these particular numbers to find a more efficient and illuminating, and less pedestrian, route to the answer. Here, because $1 \times 1 = 1$, the only sub-calculations required for $11{,}111 \times 11{,}111$ are simple additions ($0 + 1$, $1 + 1$, etc.). On the other hand, if the person already happened to have experience multiplying repdigit numbers like these, then that specific knowledge might revert the problem back to routine—they might even be able to write down the answer immediately—although not now because of their prior knowledge of long multiplication, but because of their prior knowledge of the properties of the products of repdigit numbers. They might see straightaway that the answer will be the *palindromic* (i.e., identical when read forwards and backwards) number: 123,454,321.

Just as high cognitive demand should not be conflated with non-routineness, nor should the setting of a problem in a (pseudo)-real-world context, as a so-called 'word problem'. If the context in which the task is embedded does not significantly obscure the method that the student needs to follow to tackle it, the task remains routine. For example, a task such as 'Qayla has 15 sweets and Maya has 20 sweets. How many sweets do they have in total?', while set in a real-life context, would still be, for many students, highly routine. Adding together numbers of sweets is a highly typical school mathematics pseudo-context, and indeed the use of the word 'total' in the question cues students into the appropriate operation to use (addition). There are no redundant quantities in the question, such as would be the case if, for example, the *age* of one of the children were included, and so the demands of this task for many students might be very similar to those of the task 'Find the total of 15 and 20'. For this reason, such a task would be likely to be experienced as 'routine' for many students.

From a cognitive load theory perspective (Tricot & Sweller, 2014), students would be expected to be unlikely to be successful at solving non-routine problems, since, by definition, they do not have access to the necessary schemas (Kirschner et al., 2006; Tricot & Sweller, 2014). The only way someone could solve such a problem would be inefficiently, by 'brute force' methods, such as means-end analysis or an exhaustive search through all possibilities (Newell & Simon, 1972). These approaches draw on biologically primary, domain-general skills that almost everyone acquires naturally, without being taught (Geary, 2008). Human beings have evolved to acquire biologically primary knowledge and skills, such as learning to walk and native language learning, and formal teaching of such things is unnecessary (Geary, 2008). For example, it would presumably be counterproductive to try to actively 'teach' a typically developing young child 'how to walk' by breaking down that action into a series of sub-steps, to be explicitly taught one after the other. Biologically primary skills, such as walking, are learned without needing to be taught in this kind of way. By contrast, biologically secondary skills, such as reading, *do* need, in most cases, to be taught, and would rarely be expected to be just 'picked up' by simply exposing children to a plentiful supply of age-appropriate books (Geary, 2008). Applying these categories to problem solving suggests that 'common-sense', big-picture problem-solving skills, such as searching systematically through all possible cases, may be biologically primary, and therefore do not need to be taught explicitly. This suggests that the focus of teaching problem solving should be on biologically *secondary* skills, and that 'teachable aspects of problem solving skill are entirely dependent on large amounts of domain-specific information stored in long-term memory, rather than on other factors such as domain-general skills' (Tricot & Sweller, 2014, p. 266).

In the context of problem solving, prioritizing the teaching of secondary knowledge is normally interpreted to mean that *content knowledge* (both procedural and conceptual) is the only knowledge that matters, and so that extensive content knowledge is a necessary and sufficient condition for effective problem solving. Teachers should focus on teaching mathematical content thoroughly and deeply, and the ability to solve problems *using* this content knowledge is presumed to develop naturally with exposure to carefully graded sequences of problems. However, while content knowledge is of course necessary, I argue here that it does *not* seem to be sufficient, and that *domain-specific tactics* for problem solving need *also* to be taught explicitly. The top section in Figure 1 illustrates how routine tasks (exercises) can be answered by selecting and applying relevant facts and procedures from the student's toolbox in an algorithmic manner, based on procedural rather than conceptual knowledge (see Star, 2005). By contrast, the lower section in Figure 1 illustrates how non-routine tasks (problems) require *heuristic* (as opposed to algorithmic) approaches, and how the solution, though still dependent on the application of facts and procedures from the student's toolbox, is mediated by both domain-general strategies and domain-specific *tactics*. Further explication of the distinctions between domain-general and domain-specific, and between content and heuristics, are given in Figure 2. Here, we see that typical school mathematics teaching focuses on the top right quadrant (domain-specific content knowledge).

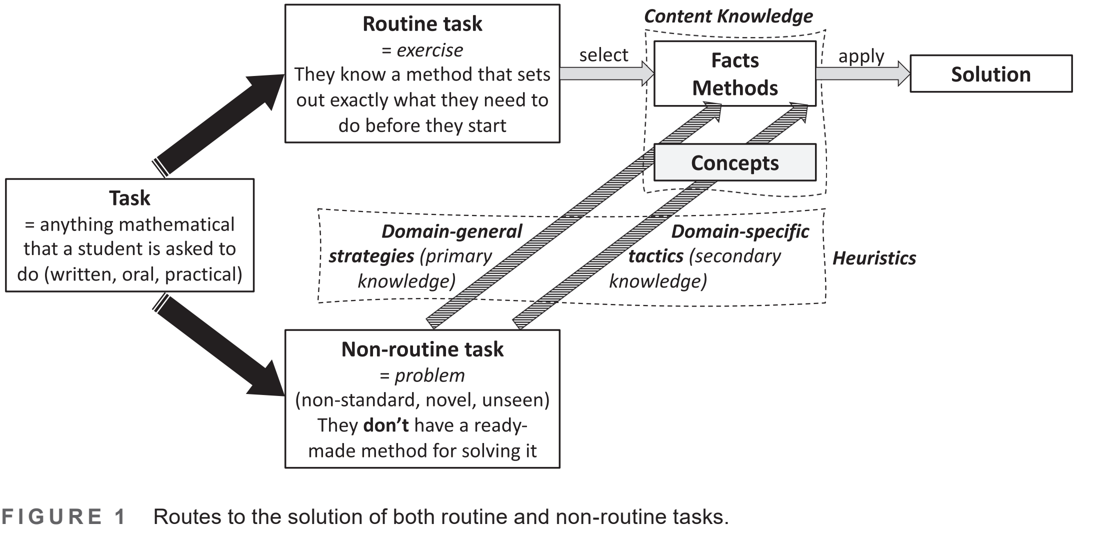


FIGURE 1 Routes to the solution of both routine and non-routine tasks.

	Domain-general	*Domain-specific*
Content knowledge	General history and philosophy of mathematics, the nature of conjecture and proof, etc.	The usual domain of school mathematics teaching: procedural and conceptual knowledge of mathematical content
Heuristics (strategies/ tactics)	These strategies cannot and do not need to be taught, because they are biologically primary	***The missing ingredient:* problem-solving *tactics*, specific to particular mathematics content areas**

FIGURE 2 The place of domain-specific problem-solving tactics.

As argued previously, when problem solving is addressed within the curriculum, this tends to be in the domain-general sense of the bottom left quadrant (domain-general heuristics). Figure 2 draws attention to the 'missing ingredient' that is simultaneously domain-specific *and* heuristic, and which I term 'domain-specific tactics'. I will provide several examples in the next section to clarify the distinctions between domain-general strategies and domain-specific tactics—the bottom two quadrants in Figure 2.

PROBLEM-SOLVING STRATEGIES AND TACTICS

While the importance of prior mathematics content knowledge for problem solving is well established (e.g. Sweller, 1988), how students can be taught to draw on this knowledge effectively, and mobilize it in novel contexts, remains unclear (e.g. Polya, 1957; Schoenfeld, 2013). Without access to teaching techniques that do this, students' mathematical knowledge often seems to remain 'inert', and students do not experience its power for tackling unfamiliar problems (Foster, 2019). It is a commonly reported experience that although the teacher might feel confident that, in terms of content knowledge, the students 'know everything they need to know' to solve a problem, yet the students nonetheless frequently do not seem able to solve the problem.

Polya's (1957) four-step model of how to problem solve (understand the problem, devise a plan, carry out the plan, look back) was accompanied by a long list of heuristics (i.e. strategies) for solving problems. These were presented as 'natural, simple, obvious, just plain common sense [stated] … in general terms' (Polya, 1957, p. 3). They were not attempts to proceduralize the process of problem solving into merely following a series of given steps. Instead, '[h]euristic strategies are rules of thumb for successful problem solving, general suggestions that help an individual to understand a problem better or to make progress toward its solution' (Schoenfeld, 1985, p. 23).

Successful problem solvers readily recognize that they deploy these strategies. However, as Alan Schoenfeld discovered, in his ground-breaking programme of research on problem solving (see Schoenfeld, 1987, 2010, 2013), Polya's strategies 'were broad and descriptive, rather than prescriptive, [and] novice problem solvers could hardly use them as guides to productive problem solving behavior' (Schoenfeld, 1987, p. 31). This was because the strategies offered were insufficiently detailed to enable students to make use of them (Schoenfeld, 1985, pp. 72–73):

> Polya's characterizations were labels under which families of related strategies were subsumed … when you look closely at any single heuristic "strategy," it explodes into a dozen or more similar, but fundamentally different, problem-solving techniques (Schoenfeld, 1987, p. 31).

An influential US *What Works Clearinghouse* practitioner guide (Woodward et al., 2012) focused on the teaching of problem solving recommended that teachers 'Expose students to multiple problem-solving strategies' and suggested that these should be explicitly taught, and Gordon (2021, p. 393) argued that heuristics that 'include tinkering, describing, taking things apart, reasoning by analogy, trial and error, etc.' should be included explicitly as a 'meta-language' (p. 394) in the presentation of mathematics, such as in textbooks. However, anyone contemplating devising a problem-solving teaching programme based around high-leverage strategies faces a bewildering number of possible strategies to focus on, each of which, as Schoenfeld remarks, splinters into dozens of others. Numerous lists of problem-solving strategies exist (Rott et al., 2021), at various points on the generic-to-specific continuum, with more general and more specific strategies often intermingled side by side (e.g. Cuoco et al., 1996; Droujkova et al., 2016; Engel, 1998; Lim, 2013; Lorenzo, 2005; Stacey et al., 1982; Zeitz, 2021).

Furthermore, there seems to be scant evidence that students improve in their problem solving by being taught *generic* problem-solving strategies (Lesh & Zawojewski, 2007; Lester, 2013; Schoenfeld, 2013; Sweller et al., 2010). In their systematic review, Hodgen et al. (2018) found nine meta-analyses concerned with problem solving in school mathematics, but most of these focused on general teaching approaches, such as inquiry-based learning, problem-based learning and guided-discovery learning. The teaching of problem-solving *strategies* tended to centre on routine 'word problems' and involve advice such as 'Read the problem. Highlight the key words. Solve the problem. Check your work' (Gersten et al., 2009, p. 1210). Similarly, Hembree's (1992) meta-analysis of 487 studies found that the vast majority addressed routine 'word problems', finding an overall effect size of 0.77, with benefits where teachers were trained in heuristics.

Explicit teaching of mathematical content and practice of methods for solving routine exercises is clearly important and necessary for building up a toolbox of essential techniques. Unless students have key mathematical facts and procedures available to them, they cannot deploy them purposefully to solve problems, and so these aspects of content knowledge certainly need to be taught. But it often does not appear to be *sufficient* for solving unseen (non-routine) problems. A classic, and extreme, example that illustrates this phenomenon is 'Langley's Adventitious Angles', which is a famous problem posed by Edward Langley in *The Mathematical Gazette* in 1922 (Langley, 1922). It involves an 80°-80°-20° triangle, and the task is to show that the angle marked x in Figure 3 is equal to 30°. This problem has been called 'The World's Hardest Easy Geometry Problem' (https://www.duckware.com/tech/worldshardesteasygeometryproblem.html)—easy, in the sense that it requires only content knowledge from elementary geometry; hard, because most people find it extremely difficult to see how to marshal those elementary facts to solve the problem. Simply 'angle chasing' around the triangle, for instance, does not lead to a solution; however, more advanced mathematics, such as trigonometry, is not necessary either.

It does not seem to be accurate to say that the solution of a problem such as Langley's depends *only* on simple facts such as that the angle sum of a plane triangle is 180°, since many people with apparently strong knowledge of and fluency with those facts seem unable to solve it. Alongside this basic domain-specific content knowledge, there appear to be *domain-specific tactics* that are necessary to know and use. These are much finer-grained than domain-*general* problem-solving strategies, such as 'Draw a diagram'. Since Langley's problem was posed, over 40 different solutions have been produced (Chen, 2019; Quadling, 1978; Rike, 2002; see also related problems in Leikin, 2001), all of which seem to rely on

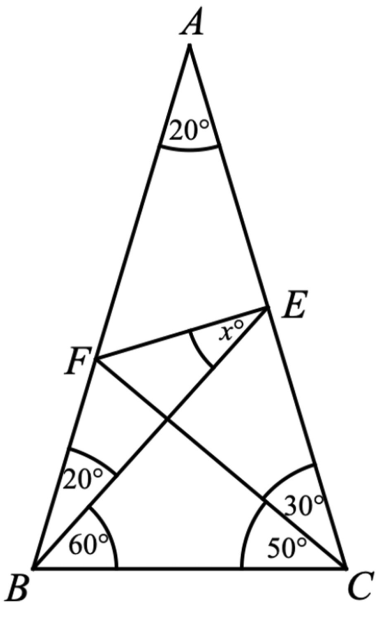


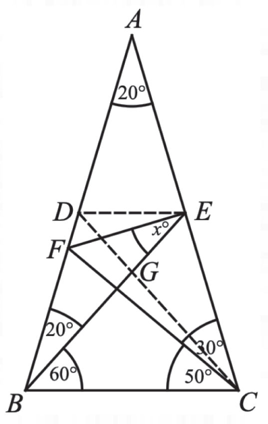
FIGURE 3 'Langley's Adventitious Angles' (Langley, 1922).

drawing in an *auxiliary line*—that is, a line that is not present in the original diagram, and which creates new configurations which allow a solution to be found (see Gridos et al., 2022).

A tactic such as 'Draw an auxiliary line' is highly domain-*specific*—it can have utility only within a narrow kind of geometry problem (Cirillo & Hummer, 2021). However, it is certainly not a standard, algorithmic procedure which can be applied mechanistically. The task is still a problem *even when the appropriate tactic is provided/revealed*, and so the tactic fails to reduce the problem to an exercise (Palatnik & Dreyfus, 2019). The problem solver still has to ask themselves: 'Where should I put the line? What should I do next to make use of this line?' For Langley's problem, shown in Figure 3, adding in the auxiliary line *DE*, parallel to *BC*, as shown in Figure 4, makes this very difficult problem suddenly solvable with elementary geometrical knowledge (Foster, 2023; Chen, 2019; Rike, 2002; see also related problems in Leikin, 2001).

It seems that tactics such as 'Draw an auxiliary line' might be the missing level of ingredient that could have the potential to transform inert content knowledge about angles into the ability to solve problems, even those as demanding as Langley's. *Without* these strategies, it can be a mystery to the teacher why a student who they are confident knows all of the 'necessary facts' (such as the angle sum of a plane triangle) could be completely stuck when faced with a problem that appears to depend mathematically only on those facts. I argue here that the answer could be that the student does not know the relevant *tactics* that would enable them to deploy those facts to solve the problem.

The relevant Polya heuristic here is the addition of an 'auxiliary element'. This strategy is much broader than drawing in an auxiliary line and is applicable in a much wider range of scenarios. It includes auxiliary *circles*, extending a line to meet some other feature, drawing a radius to solve problems involving circle theorems, and so on. At this broad level, the strategy seems too wide for the question 'When should I use this?' to have any clear answer. However, narrowing Polya's strategy to a much more specific *tactic*—'Draw an auxiliary line'—seems a potentially more appropriate grain size for students to engage with. So, here I distinguish a *strategy*—a big-picture goal and intentions—from a *tactic*—specific, concrete



Add an auxiliary line DE (shown dashed), passing through point E and parallel to BC.

Join point D to point C (line CD also shown dashed), creating a new point, G, where the line CD intersects the line BE.

$\angle BED = 60°$ (alternate to $\angle CBE$).

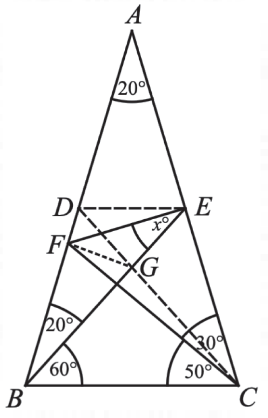
Because ΔBEC and ΔCDB are congruent (SAS), $\angle CDE$ is also 60°.

So, ΔDEG is equilateral.

Since $\angle BGC = \angle DGE$ (vertically opposite), ΔBGC is also equilateral.

Now notice that $\angle BFC$ is 50° (angle sum of ΔBFC), meaning that ΔBFC is isosceles, with $BF = BC$.

Since ΔBGC is equilateral, this means that $BF = BC = BG$.



Now add the line FG (shown dotted) to make the triangle BFG.

Since $BF = BG$, this triangle must be isosceles, with $\angle BFG = \angle BGF = 80°$, from the angle sum of ΔBFG.

We can now find $\angle DGF$ as $180° - \angle DGE - \angle BGF$ $= 180° - 60° - 80° = 40°$ (angle sum on a straight line).

Since $\angle BFG = 80°$, this must be the sum of $\angle DGF$ and $\angle FDG$ (exterior angle of ΔDFG is equal to the sum of the two opposite interior angles), so $\angle DGF$ must be 40°, meaning that ΔDFG is also isosceles, with $DF = FG$.

Therefore, ΔDEF and ΔEGF are congruent (SSS).

This tells us that the required angle x must be half of ∠DEG, which is 60°, so $x = 30$.

FIGURE 4 Solution of ‘Langley's Adventitious Angles’ by adding an auxiliary line *DE*, parallel to *BC*.

steps to achieve something. Note again, that a tactic is still *heuristic* rather than algorithmic, as students, when using it, need to employ their judgement and consider questions such as:

1. Which kinds of problems do/do not benefit from adding an auxiliary line?
2. How many auxiliary lines might be needed?
3. Where should it/they go?
4. What should be done afterwards to make use of this/these auxiliary line(s) to solve the problem?

Hsu and Silver (2014) noted that:

> the work of drawing auxiliary lines on a diagram is usually highly demanding because it forces students to anticipate the creation of subconfigurations associated with corresponding geometric properties that can be used to form a solution plan. Such anticipation further requires students to view the diagram dynamically and apply the transformational observation to visualize a solution that can be generated with the help of such auxiliary lines (p. 472)

As stated above, being given the tactic does not reduce the problem to a routine exercise; instead, it allows students to develop a sense of ‘how, why, and when they can transform a diagram in a proof’ (Senk, 1985, p. 455). Students who have not explicitly been taught such tactics may even feel that they have never been given 'permission' to make alterations such as this to a given diagram as part of their solution. Merely being provided with the language of there being such a thing as an 'auxiliary line' may unlock possible fruitful approaches for the student.

EXPLICIT TEACHING OF PROBLEM-SOLVING TACTICS

Langley's problem, mentioned above, is highly demanding and unlikely to be well suited for use in most secondary school classrooms. I introduced it here in order to show how difficult a problem can be when the required tactic(s) are not available, even when the necessary content knowledge that the problem requires is very simple. However, the tactic of 'Draw an auxiliary line' has very wide applicability to non-routine 'puzzle' problems in geometry (e.g. Shearer & Agg, 2019). A much simpler and more suitable problem for introducing the tactic to secondary-age students could be the one shown in Figure 5a (taken from Baldry et al., 2023). Baldry et al. (2023) analysed a Japanese problem-solving lesson, based on the problem shown in Figure 5a, in which the teacher taught *explicitly* about the use of auxiliary lines. Auxiliary lines such as those shown in Figure 5b,c lead to simple, one-step solutions based on elementary angle properties (such as the angle sum of a plane triangle being 180°, the base angles in an isosceles triangle being equal), whereas an auxiliary line such as the one shown in Figure 5d does not.[1]

In the lesson described in Baldry et al. (2023), students first attempted to solve the problem *without* being explicitly taught the tactic of 'Draw an auxiliary line'; then, they explored the potential of the tactic. In subsequent lessons, students might be invited to articulate the features of a problem that would make an auxiliary line a productive approach, and they might be invited to sort a range of problems into those that could benefit from introduction of an auxiliary line and those that would not. They might be invited to pose some problems, as different from one another as possible, that can all be solved by using an auxiliary line. They might also be asked to invent a problem that *looks as though* it might be solved by using an auxiliary line, but where this approach fails, and to explain why. This example illustrates a possible general approach that could be broadly applicable to teaching a domain-specific problem-solving tactic that generalizes across a relatively narrow range of (geometrical, in this case) content. This kind of approach is prized within the tradition of the Japanese problem-solving lesson, and the 2019 Trends in International Mathematics and Science Study[2] reported that Japanese students were much better than English students at solving mathematical problems (Mullis et al., 2020). Given that the Japanese problem-solving lesson is widely valued and taught within Japan, and focuses on teaching students to solve mathematical problems, it seems likely that this may be part of the reason for the success of Japanese students in this area.

There is space here to give one further example of the teaching of a domain-specific, problem-solving tactic that falls under one of Polya's overarching strategies. One of the broadest of Polya's strategies is 'Draw a diagram', popularly implemented in classrooms in some parts of the world currently as 'If it's tricky, draw a piccy!' As with the other strategies, successful problem solvers do indeed often benefit from utilizing well-chosen diagrams. However, faced with the advice 'Draw a diagram', the novice is likely to ask, 'What diagram should I draw?' A student presented with a mathematics problem about cars might, perhaps in desperation, sketch a car, but this may well be of no help to them in solving the problem. In the context of a routine exercise, such as a standard mechanics question, for instance, the instruction to 'Draw a diagram' may be perfectly clear: the student is intended to sketch a 2D representation of the physical situation described in the question, making simplifying assumptions as they go, such as using perfect vertical lines for walls and circles for balls, and indicating relevant distances, forces, velocities, etc. by using arrows. However, for a

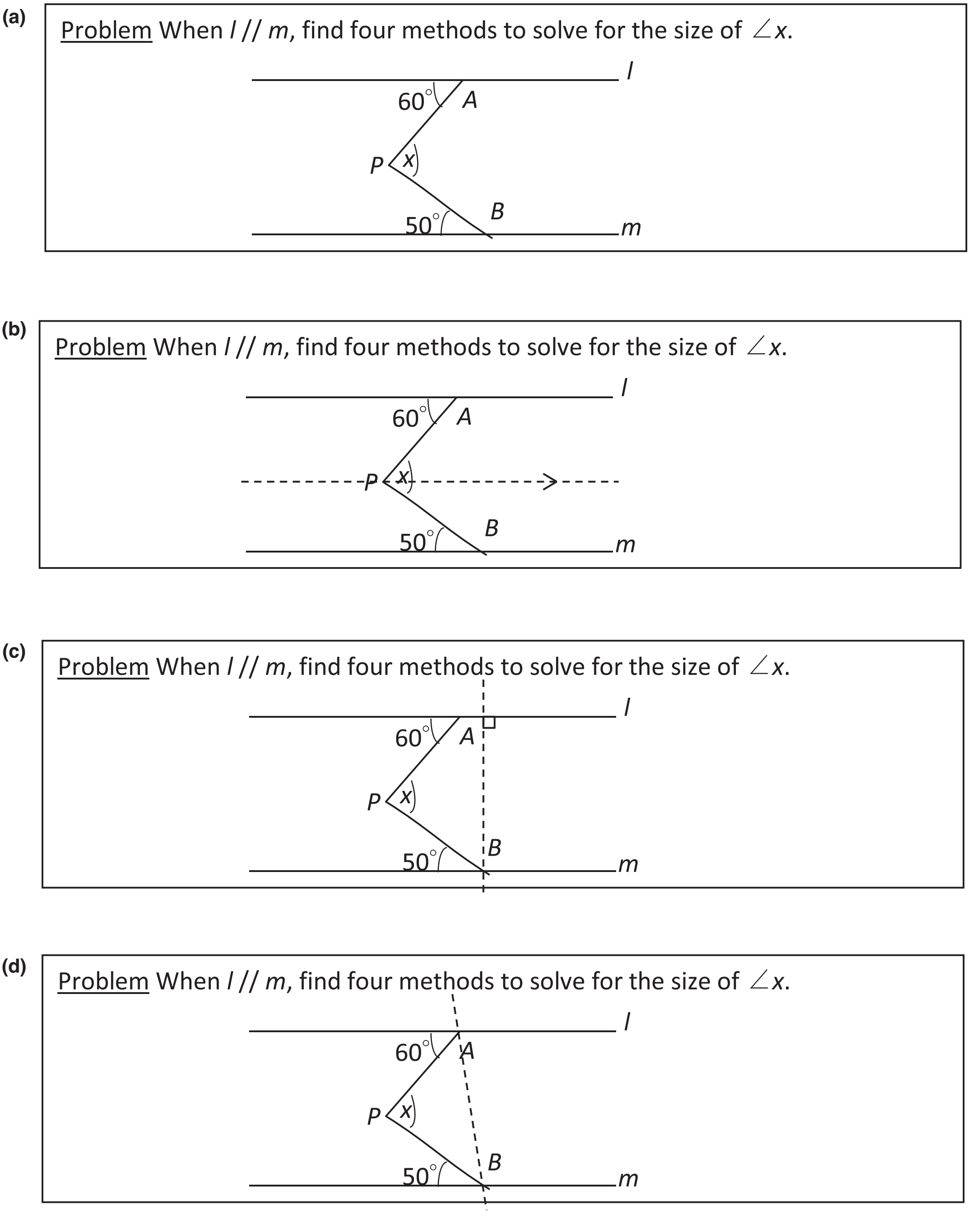


FIGURE 5 Auxiliary lines (shown dashed) that enable (b, c) or do not enable (d) a simple one-step solution to the problem shown in (a) (adapted from Baldry et al., 2023). Note that the notation '*l* // *m*' means that 'line *l* is parallel to line *m*'.

non-routine problem such as we are considering here, the kind of diagram that might assist is likely to be much less obvious.

To narrow Polya's domain-general strategy 'Draw a diagram' to a more domain-specific *tactic* that can be taught explicitly, a particular area of mathematical content needs to be selected. Once problems are restricted to a narrow area of content, there is no longer a functionally infinite number of possible diagrams that are might be helpful, and often there would seem to be only a relatively small number of possibilities. For example, diagrams are frequently deployed to solve probability problems, and, within this content domain, we might subdivide the generic strategy 'Draw a diagram' into the following three alternative *domain-specific tactics*:

1. Draw a Venn diagram.
2. Draw a tree diagram.
3. Draw a Carroll diagram (i.e. a 2-way table or contingency table).

For example, the problem 'Find the probability that a randomly-chosen integer between 1 and 15 inclusive is both greater than 7 and a multiple of 5' might be solved using a Venn diagram (Figure 6a). A different kind of problem, such as 'Find the probability of obtaining a total of 10 when throwing two fair dice' might be solved using either a tree diagram (Figure 6b) or a Carroll diagram (Figure 6c). These diagrams are all commonly taught within the school mathematics topic of probability, but their different advantages and disadvantages, and the kinds of problems for which each diagram is particularly suitable, is perhaps rarely taught explicitly. Some problems might perhaps be solved equally well by using any of these three kinds of diagram; other problems *might* be solved using any of the diagram types, but one diagram type has a clear advantage over the others; and other problems may realistically be solved only by using one of these diagram types. Students might explore a variety of probability problems and work on tasks such as:

1. Can you make up two similar-looking problems, one which can be solved with a Carroll diagram and one that cannot? If so, say which is which and explain why.
2. Can you make up a problem which can be solved equally well by using a tree diagram or by using a Venn diagram?
3. (How) can you identify *mutually exclusive* events when using each of the three diagrams?
4. (How) can you identify *independent* events when using each of the three diagrams?

Various benefits may become apparent through a lesson with this kind of focus. For example, consider the two superficially similar questions:

1. I throw *two* fair dice. What is the probability of obtaining a total of 10?
2. I throw *three* fair dice. What is the probability of obtaining a total of 10?

The first question may be solved by using either a tree diagram (Figure 6b) or a Carroll diagram (Figure 6c), whereas a Carroll diagram is difficult to use for the second question, because in two dimensions only the outcomes from two dice can be shown simultaneously, and so six separate Carroll diagrams would be needed (Figure 7a). On the contrary, a tree diagram can readily accommodate three dice (see Figure 7b), although this entails $6^3 = 216$ branches, which are hard to discern individually in Figure 7b. These pros and cons and trade-offs seem important for students to grapple with. When different kinds of diagrams are taught sequentially, without the kind of comparing and contrasting outlined here, students may develop simplistic global preferences for one kind of diagram over another (e.g. 'I don't like Venn diagrams') and fail to appreciate that the nature of the particular problem should determine which kinds of diagrams are likely to be more or less helpful.

Based on the kinds of approaches seen in Japanese problem-solving lessons (e.g. Baldry et al., 2023), a possible approach to teaching a domain-specific problem-solving tactic could involve proceeding through the following six steps, although it is important to

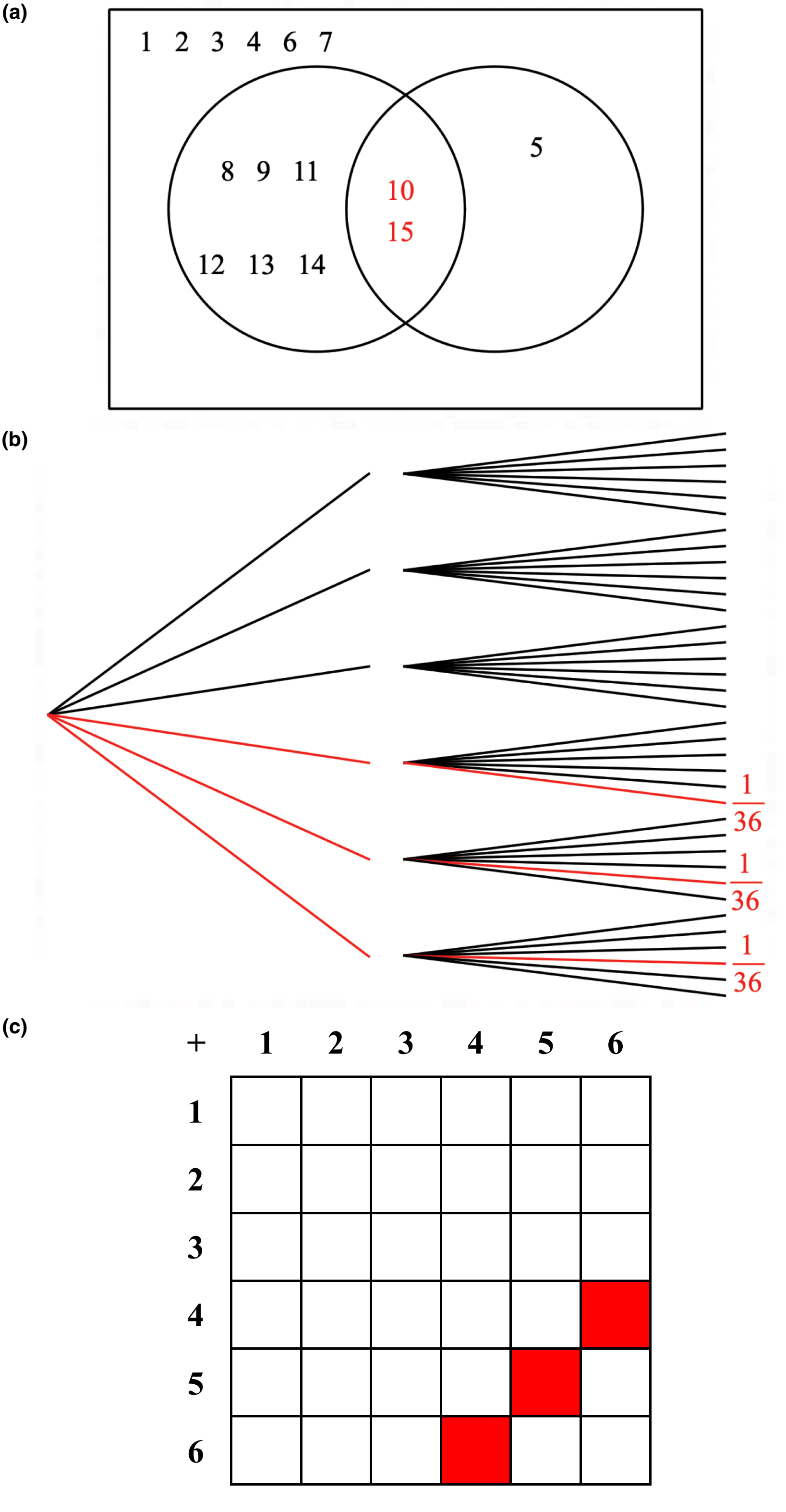


FIGURE 6 Solving probability problems using (a) a Venn diagram, (b) a tree diagram and (c) a Carroll diagram.

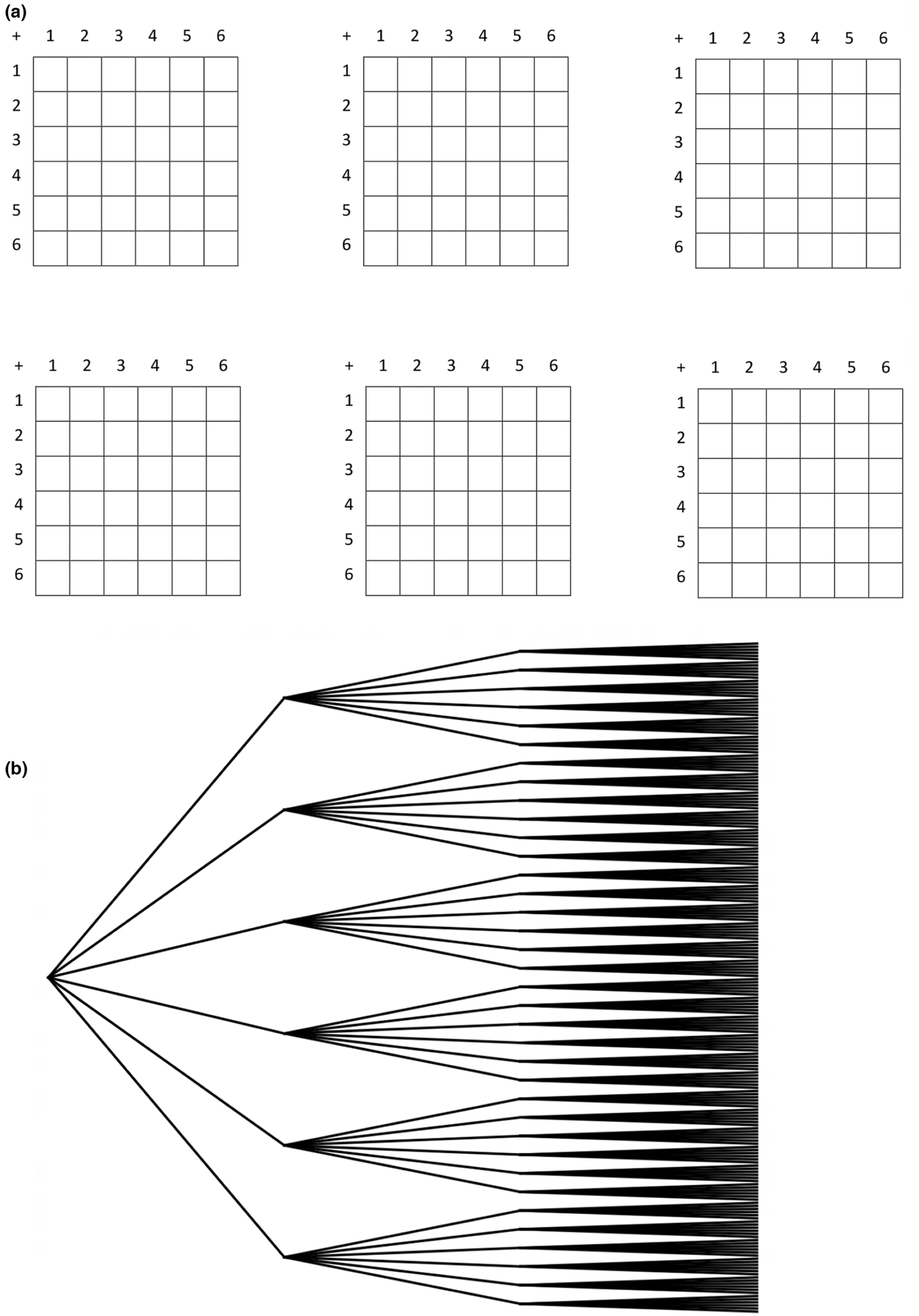


FIGURE 7 Answering the question 'I throw three fair dice. What is the probability of obtaining a total of 10?' using (a) Carroll diagrams; (b) a tree diagram.

note that these are merely presented as one possible approach, and other models would also be possible:

1. Choose an important, domain-specific problem-solving tactic.
2. Find a problem that is dramatically unlocked by using that tactic and which depends only on mathematical content already familiar to the students.
3. Initially, allow students to struggle with that problem first (without the tactic being explicitly presented or mentioned).
4. Then, introduce the tactic and allow the students to succeed in solving the problem by using it.
5. Explore through discussion the features of the problem that made that tactic such a good fit.
6. Present (and invite students to construct) other problems that do and do not (less obviously and more obviously) benefit from use of the same tactic and explore why.

Over time, as students build up a toolkit of domain-specific tactics, problems may be interleaved which, while looking superficially similar, benefit from different tactics. Conversely, problems might be offered which look superficially different but can nevertheless be solved using the same tactic. Problems which may be solved using multiple contrasting tactics might be introduced, so that students can compare and contrast completely different solution approaches.

In this way, it is envisaged that students build up a comprehensive set of problem-solving tactics—a toolbox of carefully selected tactics—which enable them to make powerful use of the mathematical content knowledge which they are learning in parallel with this. In addition to amassing tactical knowledge, students develop experience regarding when particular tactics are more or less likely to be relevant, so that over time they become able to select efficiently an appropriate tactic to meet the needs of a particular problem. A well-chosen set of problem-solving tactics, taught explicitly and systematically, is intended to equip students to be powerful solvers of mathematical problems.

CONCLUSION

It is often claimed that 'The most effective way for students to learn to solve problems is for them to solve a variety of problems' (Lester & Cai, 2016, p. 121). However, this approach risks a *Matthew effect* (Merton, 1968), in which 'the rich get richer and the poor get poorer'. Here, students who happen to discover productive strategies quickly for themselves rapidly accelerate away from other students who, for whatever reason, do not. Leaving strategies and tactics to be *discovered* in an ad hoc fashion does not seem to promote equitable mathematics education (Kirschner et al., 2006). Students with more advantageous socio-economic backgrounds are more likely to have received enhanced opportunities for support at home to develop dispositions that allow them to make good use of open-ended opportunities, which other students may simply struggle with and learn very little from. 'Teaching mathematics through problem solving', while an excellent approach to teaching mathematical content, does not seem to guarantee that all students will develop the necessary toolbox of *specific* tactics that will enable them to tackle a wide range of unseen problems confidently and reliably, and risks leaving content knowledge inert. There would currently seem to be an extreme imbalance in the quantity of resources typically invested in teaching mathematical *content*, as compared to those directed towards helping students to *make use* of that content to solve problems. Allowing problem solving to be absorbed into the content curriculum as 'teaching mathematics through problem solving' seems to risk preparation for problem solving disappearing altogether.

The overarching question for this paper was: *How can the explicit teaching of domain-specific tactics, rather than domain-general strategies, within the mathematics curriculum enable all students to become powerful problem solvers?* The approach advocated in answering this question may be captured in the following two claims:

> *Claim* 1. There are well-defined, domain-specific problem-solving tactics that can enable all students to productively tackle non-routine problems within any specified content area.

> *Claim* 2. Tactics such as these can be (but rarely are) taught explicitly in a systematic fashion and in a carefully planned sequence.

I have argued for Claim 1 in the Problem-Solving Strategies and Tactics and Explicit Teaching of Problem-Solving Tactics sections of this paper. I have suggested in the Explicit Teaching of Problem-Solving Tactics section that Claim 2 may be true, but further work is needed to develop such a teaching programme based around this approach and demonstrate its effectiveness in enabling students to become powerful problem solvers. In this paper, I have sought to argue for the plausibility of Claim 2, acknowledging that we do not yet have any empirical evidence to support it. This work is underway in the curriculum design work in which my collaborators and I are currently engaged (Foster et al., 2021).

Currently, school 'problem-solving lessons' often exist as standalone, one-off lessons, intended by their designers as a 'dietary supplement' to the usual curriculum, essentially interchangeable, and with the intention that the more of these that might be experienced the better (see Foster, 2019). They are rarely sequenced into a coherent, problem-solving curriculum that systematically addresses important, high-leverage tactics, teaching these in a well-planned order, with appropriate review and consolidation. Breaking down some of the highest leverage of Polya's strategies into detailed, actionable, domain-specific tactics, focused on well-defined content areas, seems to have the potential to mobilize students' potentially inert content knowledge in the service of powerful problem solving of non-routine problems.

One possible objection to the approach presented here might be that the teaching of problem-solving tactics is 'just good content teaching'. This may indeed be the case in some places (e.g. in the best cases showcased in Japan, see Baldry et al., 2023). However, it seems that such approaches are uncommon in the West, where most school mathematics students might be expected, for example, to be unfamiliar with a term such as an 'auxiliary line' or be able to say clearly when they would choose to use a tree diagram in preference to a Venn diagram. Shifting the focus from domain-general strategies to biologically secondary, domain-specific tactics—distinct from ordinary mathematical 'content'—seems to offer the potential to equip students far better to become powerful problem solvers.

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The author reports that there are no competing interests to declare.

DATA AVAILABILITY STATEMENT

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ENDNOTES

[1] It is possible to use the auxiliary line shown in Figure 5d to obtain a solution if a new unknown (say, y) is introduced. Angle BAP is labelled as y, and this means (by the supplementary property of co-interior angles) that $\angle ABP = 180 - 50 - 60 - y$. Now, using the fact that the angle sum of triangle APB is $180°$, we obtain that $x = 110°$. However, while this is possible, it is considerably more involved than the approaches based on the auxiliary lines shown in Figure 5b,c.

[2] TIMSS (Trends in International Mathematics and Science Study) is an international comparative study that takes place every 4 years. It focuses on students around the world aged 9–10 and 13–14 and considers their knowledge and the characteristics of their teaching.

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