Understanding the Discipline of Mathematics

A guide to Disciplinary Literacy
in the Mathematics classroom

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Contents

Acknowledgements

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Mathematics literacy: an introduction

What does it mean to read mathematics?

When thinking about the question of 'why disciplinary literacy for mathematics?', it's worth considering one of the enduring challenges in the teaching of mathematics: supporting our pupils to think in the expert ways that we, as their teachers, do.

In other words, how can we help pupils be as 'literate' in mathematics as we are? This is perhaps slightly easier in mathematics as it could be said the question of 'what makes an expert mathematician?' is less controversial than in other subjects: someone who can take mathematical knowledge and consistently apply it in familiar, and unfamiliar, contexts; someone who is highly literate in mathematics.

However, a more challenging question is how we create and develop highly literate mathematicians, and this is where we might consider disciplinary literacy. Disciplinary literacy can present some solutions to the challenge of building expertise and making our pupils literate in mathematics ie it can help our pupils take existing mathematical knowledge and apply it in new contexts.

What is the aim of the resource?

This resource aims to act as a guide for colleagues looking to tackle some of the common challenges in the mathematics classroom. Challenges including:

- *How can we help pupils make sense of new information?*
- *How can we help pupils connect new information to existing knowledge?*
- *How can we support pupils when solving novel problems?*
- *How can we support pupils with worded problems?*

It looks at the challenges through three lenses unique to the discipline of mathematics:

- **Mathematical vocabulary**
- **Mathematical sense-making**
- **The importance of prior knowledge**

This resource attempts to synthesise general and subject-specific research, recognising that disciplinary literacy is '*general and subject-specific*'. The clearest example of this is how we've adapted the 'Big Five' Reading Strategies recommended in the EEF's *Secondary Reading Guidance Report* (2019), by prioritising:

- **Activate prior knowledge**
- **Make predictions**
- **Clarify uncertainty**

We have subsumed 'Generate questions' into 'Clarify uncertainty' – we believe asking questions and seeking answers work in tandem when approaching a worded problem. Rather than 'Summarise', we will outline the strategy of 'Translation' – this is a term better understood from a Mathematical perspective, therefore offering a disciplined focus on a more generic reading strategy.

We feel this enables us to provide clearer focus for other teachers of Mathematics. Putting this together has helped us find common ground across subjects and to support one another in our thinking. It's also something we believe will help school leaders implement disciplinary literacy on a departmental and whole school level.

Mathematics literacy: an introduction

How might you use our resource?

We have structured the guide into two main parts:

1. *Exploring Mathematics as a language and the role of Disciplinary Literacy in helping pupils' understanding*
2. *How-to guides for effective use of Disciplinary Literacy in Mathematics*

1 Exploring Mathematics as a language and the role of Disciplinary Literacy in helping pupils' understanding

In this section, we start by describing the discipline of mathematics as a reminder of the purpose and construct of our subject. We consider what it means to read mathematics and how expert mathematicians read their subject. By considering the 'habits' of mathematicians, we can consider the strategies we want to teach our pupils.

We go on to look at the unique challenges of reading in mathematics as outlined above: vocabulary, sense-making, and the importance of prior knowledge.

Grounding ourselves with a solid understanding of the barriers to pupil literacy in mathematics allows us to better diagnose the issues when they arise, and to select the most appropriate response or strategy to support. In this section, we discuss two strategies in particular that can support pupil understanding: *Translation* and *Close Reading.*

2 How-to guides for effective use of Disciplinary Literacy in Mathematics

Having looked at the challenges in the previous sections we then contextualise the solutions we discuss. In this section, we focus on two specific strategies: *Using Think Aloud* and *An approach to worded problems*. Both of these build on the content covered previously, including the two reading strategies and the adapted strategies taken from the EEF.

Our main goal is to support pupils to overcome the barriers they face, and identified in the first section. For example, *Translation* can help a pupil change a mathematical concept from one format to another (eg from an equation/prose to a graph), but using a *Think Aloud* can support pupils to understand *why* doing that might be helpful. Doing this can support them to recall this technique in future situations where it might be helpful, and execute it successfully.

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Part 1

Exploring Mathematics as a language and the role of Disciplinary Literacy in helping pupils' understanding

What is the discipline of mathematics?

Mathematics is the language by which we represent the natural laws that govern the universe. It's the language that allows us to understand our experiences and create new ones through the constant development of technology.

Mathematicians:

- **study numbers, shapes and patterns**
- **formulate new conjectures**
- **establish truth by logical reasoning**

Over time, mathematics has evolved via abstraction and logical reasoning from counting, calculation and measurement. Practical mathematics has been a human activity since written records existed. Rigorous mathematical arguments in the West first appeared with Euclid, considered the 'father of geometry' in Euclid's Elements.

Today, mathematics is used as an essential tool across many fields and disciplines such as engineering, natural sciences, finance, computer science, medicine and social sciences.

Mathematics is often seen to have two separate strands[1]:

Pure	Applied
Pure (theoretical) mathematics – mathematics for its own sake without having any application in mind (eg prime numbers or algebraic expressions).	Applied (practical) mathematics – the application of mathematical knowledge to other fields (eg physics and chemistry).

While these strands may be seen as separate, it's often found that practical applications for what began as 'pure mathematics' are discovered later.

In schools, mathematics is commonly broken down into:

- **number**
- **algebra**
- **geometry**
- **ratio and proportion**
- **statistics**

Through these key strands, pupils learn mathematical skills crucial to everyday life and society, and learn the skills needed to understand the contents of other school subjects including science, social studies, music and art.

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1. Fang, Z. (2023). *Demystifying Academic Reading.* Taylor & Francis.

What does it mean to read mathematics?

> **"The language of mathematicians is precise, one in which every word is included deliberately and slight changes will alter the meaning of an entire proposition"[2]**
> Shanahan & Shanahan, 2008

Mathematics is a unique subject as it seems to have its own language, communicated through the use of words, symbols, diagrams and equations. Mathematicians will use diagrams, symbols, numbers and shorthand. Reading mathematics requires interpretation, translation and pattern recognition; and, to add another layer of complexity, when we're reading questions that involve something in addition to words, like diagrams, statistics or equations, we don't necessarily read from left to right or top to bottom. We jump around the page based on the information we've read and how it links to the diagram, for example.

To be able to fluently read mathematics we need specialist mathematical knowledge to be able to understand, interpret and take meaning from what is being written and what is being asked of us. This is where disciplinary literacy comes in; **mathematical disciplinary literacy is about supporting pupils to be literate in the unique ways of thinking and communicating in mathematics.**

For example, if an expert mathematician was asked "*find the longest side of a right angled triangle that has perpendicular lengths of 3cm and 4cm*", they're likely to immediately start thinking of Pythagorean triples. Without this specialist background knowledge however, it's much more difficult to read and understand what's in front of us; indeed it's likely that question would be just an abstract statement devoid of much meaning.

How do mathematicians read mathematics?

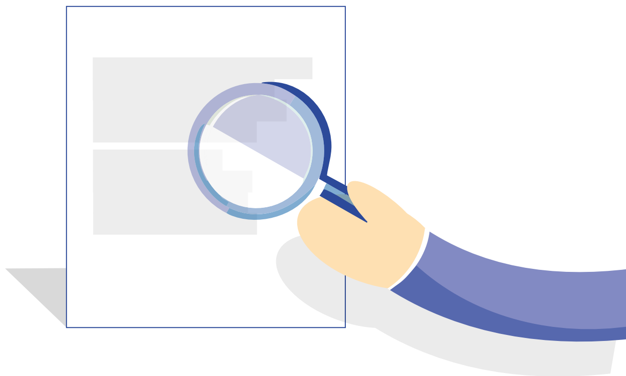
When mathematicians are reading, they use information to:

- **look for patterns and relationships**
- **decipher symbols and abstract ideas**
- **ask questions**
- **apply mathematical reasoning**
- **piece together a solution**

Mathematicians read and make sense of mathematical information in a number of ways, whether that's presented using words, diagrams, symbols, or otherwise. **They employ specific strategies to help make sense of information**, and in this document, we will consider two of these: **Close Reading** and **Translation**.

The first of those, **Close Reading**, *"is an intensive analysis of a text in order to come to terms with what it says, how it says it, and what it means".*[3]

This often involves multiple re-readings to pick out key information. In mathematics, it's likely that these re-readings may not happen from start to bottom, or after getting through to the end and then starting from the beginning. As mentioned, mathematicians are likely to read through the information in its entirety first, before scanning back and forth to identify what information can be linked and how.



2. Hillman (2013). *How do secondary school teachers apprentice students into mathematical literacy.*
3. Shanahan, T. *What is close reading?* shanahanonliteracy.com/blog/what-is-close-reading

Consider the example below:

There are 3 counters in a bag. One counter is red. One counter is green. One counter is blue. Mike takes at random a counter from the bag. He puts the counter back in the bag. Then Ellie takes at random a counter from the bag. Is Ellie more likely to take a blue counter from the bag than Mike? You must explain your answer.

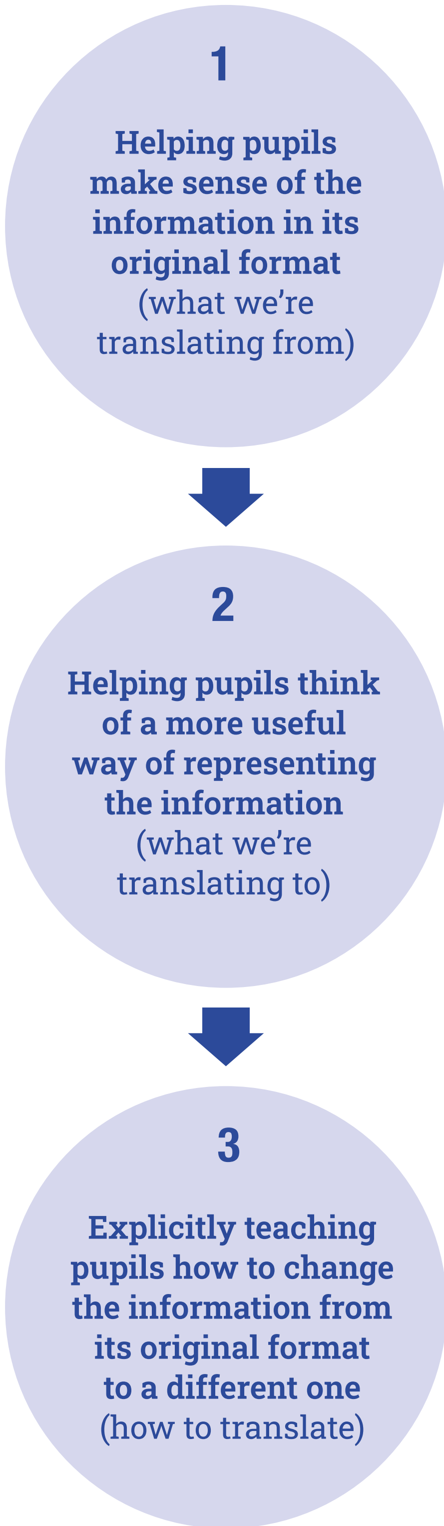
With this question, it's likely that an expert mathematician employing a **Close Reading** strategy would first read through the entire question to understand the situation and end goal, before re-reading the problem to sort the relevant information from the irrelevant.

The second strategy, **Translation**, *"is a process in which constructs of one mathematical representation are mapped onto those of another"* [4]. In other words, it involves taking a problem and converting it into a different, but equivalent form.

Translation is something mathematicians must often think about and its importance is widely agreed[5]. Examples can include verbal to symbolic[6] (eg forming a simultaneous equation from a word problem), symbolic to graphical (eg taking an equation and presenting it as a graph), or verbal to graphical (eg constructing a bearings diagram from a worded problem).

While the descriptions we've offered here seem simple, the ability to do this successfully is a different matter altogether. Translation requires not only a knowledge of the current format, but also knowledge of a different format that might be more meaningful. To do this successfully requires a significant amount of prior knowledge and, crucially, support in applying the prior knowledge to the new situation. This will often involve explicitly teaching pupils[7] how to do that, such as modelling this process or using a worked example. Tying these threads together, we can increase the likelihood of successful translation by following the three-step process in figure 1.

Figure 1

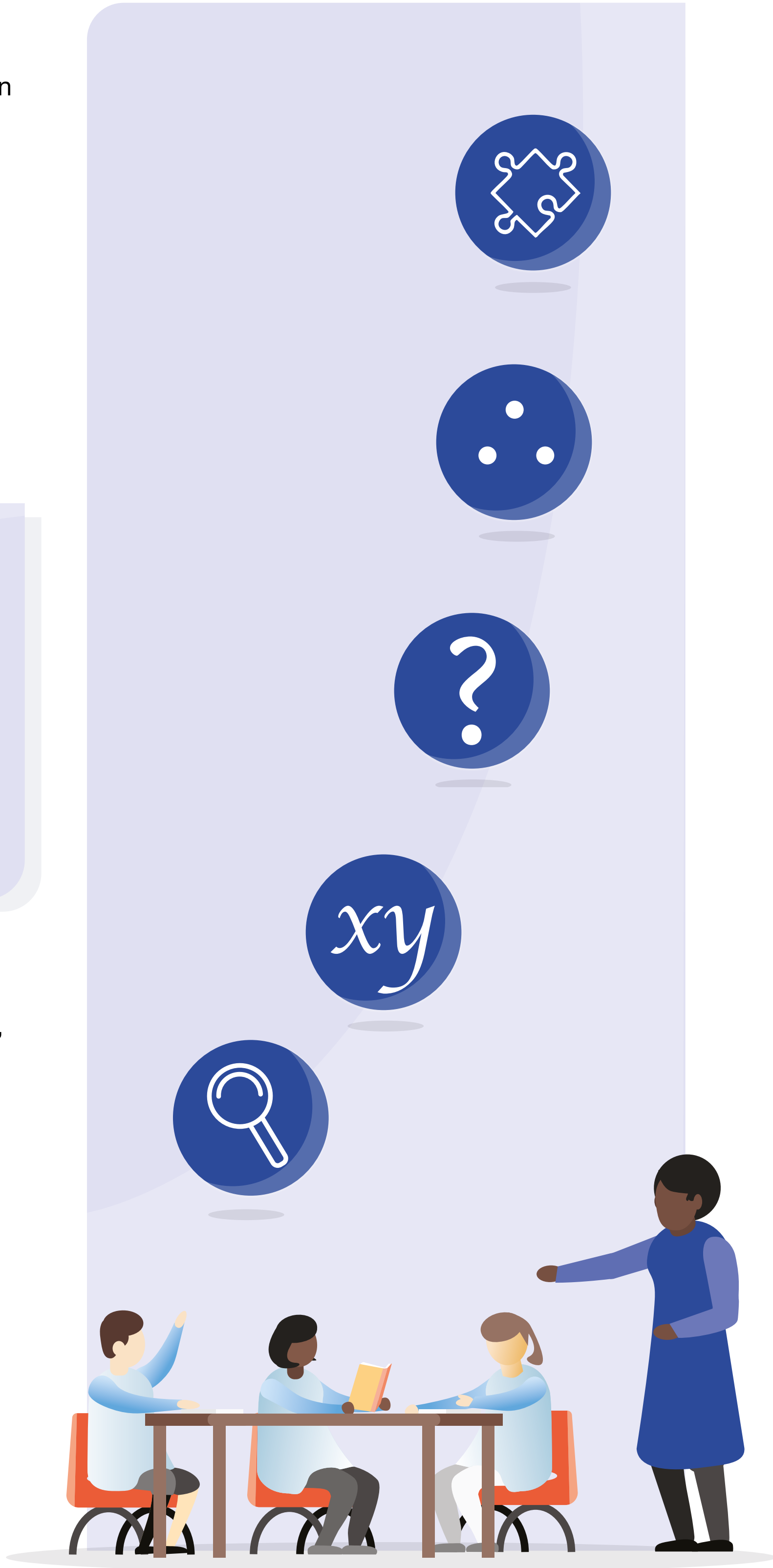


4. Janvier (1987), as cited in Bossé, M. J., Adu-Gyamfi, K., & Cheetham, M. (2011). *Translations Among Mathematical Representations: Teacher Beliefs and Practices*. International Journal for Mathematics Teaching & Learning.
5. Bossé, M. J., Adu-Gyamfi, K., & Cheetham, M. (2011). *Translations Among Mathematical Representations: Teacher Beliefs and Practices.* International Journal for Mathematics Teaching & Learning.
6. ibid
7. Wooldridge, C. and Weinstein, Y. (2016). *What's Transfer, and Why is it so Hard to Achieve? (Part 2). The Learning Scientists*. learningscientists.org/blog/2016/6/9-1

Research in this domain underscores the difficulty of this, with many studies suggesting that pupils tend to perform worse on word problems than when the same problem is presented mathematically[8], a finding which chimes with many of our own classroom experiences.

While this can often be a frustrating experience for us as teachers (because we know they can 'do the maths'), it also highlights the impact that having the ability to translate between different forms of presenting information in mathematics can have:

By helping pupils to translate accurately and appropriately, we're making it more likely that they're able to solve problems that would otherwise be inaccessible to them for reasons other than their pure mathematical knowledge.



A final point is the ability to read and make sense of mathematics is closely linked to problem solving, as the examples later in this document will demonstrate. Problem-solving is a key skill we hope to develop for our pupils and a significant challenge, but research suggests that the better pupils are able to make sense of the problems in front of them (through strategies such as **Close Reading** or **Translation**), the more likely they are to be able to answer them successfully.[9]

8. Huang, J. & Normandia, B. (2008). *Comprehending and solving word problems in mathematics: Beyond key words*. In Z. Fang & M.J. Schleppegrell (Eds.), *Reading in secondary content areas: A language-based pedagogy* (pp. 63–83). Ann Arbor, I:University of Michigan Press
9. ibid

The challenges of reading in mathematics

> **"Much of math's precision comes from the use of symbolic notation to express quantities, functions, and operations in the fewest possible terms, a form of text uncommon in other disciplines."[10]**
> Kintsch

Vocabulary

Mathematical vocabulary can be confusing for pupils as they begin to read mathematics because they try to relate mathematical language to the English language. Some of these complexities are summarised below[11]:

- Words occur in Mathematics and English but they have distinct meanings eg power, origin, prime, even, series
- Words occur in Mathematics and English that have comparable meanings but when reading Mathematics it is much more precise eg similar, average, reflection
- Words occur only in Mathematics eg hypotenuse, polygon, asymptote.

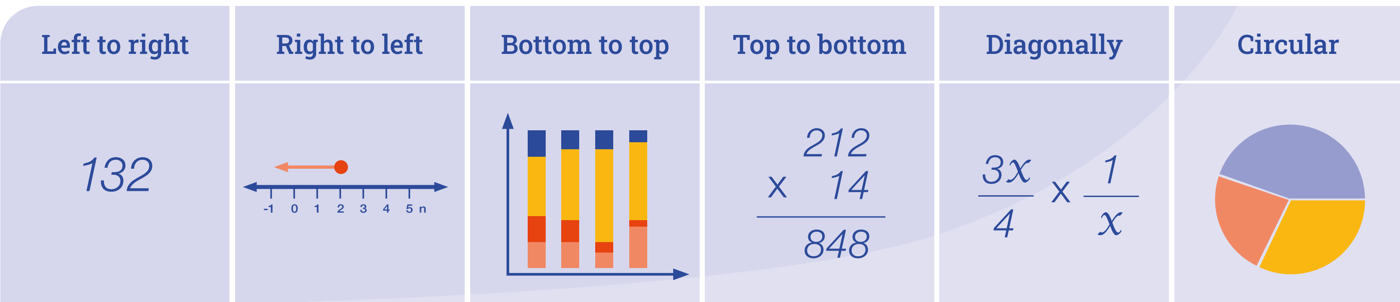
To layer more complexities onto mathematical vocabulary we can also find some mathematical words have more than one meaning eg base (*eg working in base 10 vs the base of a shape), degree* (*eg the size of an angle vs the order of a polynomial*), inverse (*eg the inverse of an operation vs the inverse of a function*), or they have a different meaning in another subject eg solution (relevant in chemistry) or subject (relevant in English)[12].

Which order to read in?

In addition to the challenge presented by vocabulary, there's also the challenge of how to make sense of the information presented to us. In nearly all other subjects we teach pupils to read from left to right. However, in mathematics there are scenarios where we will read top to bottom, left to right, right to left, bottom to top and in some cases diagonally or circularly.[13]

The examples in figure 2, below, illustrate how mathematicians might read the information presented (*caveat: while some of the examples will be exclusively read in the manner described [eg 132], others can be read more than one way and they will often involve re-readings that incorporate a variety of directions [eg the multiplication or fractions examples]*).

Figure 2



10. ibid
11. Thompson, D. R., & Rubenstein, R. N. (2000). *Learning Mathematics Vocabulary: Potential Pitfalls and Instructional Strategies*. *The Mathematics Teacher*, 93(7), 568–574.
12. ibid
13. Nolan, J.F. (1984) *Reading in the content area of mathematics*. In *Reading in the Content Areas: Research for Teachers*. Newark, Delaware: International Reading Association, 28-41.

Reading in Maths can be non-linear or multi-modal

There's a lot of non-linear reading in mathematics, when a reader must cross reference between different parts of texts and/or diagrams and simultaneously eliminate information/sentences that are unnecessary because maybe they are already shown in a diagram.

Items that are represented in similar ways visually can often convey completely different meanings. For example, 3½ and 3n both have the number '3' with an item immediately next to them but communicate completely different mathematical operations (addition and multiplication respectively).

Every sentence matters

A final challenge is that the way that mathematicians make sense of information fundamentally differs from other disciplines and acknowledging this difference is something pupils often fall foul of not doing.

As noted by Shanahan and Shanahan (2008), *"Students often attempt to read mathematics texts for the gist of general idea"* (p49).[14] This strategy may work in other disciplines eg when attempting to find the main themes from a news article, but will very often fail in mathematics due to the deliberate nature of each bit of text or information.

Research around 'key word' strategies demonstrates this point nicely. Even when pupils are taught a 'key word' approach in mathematics (eg teaching pupils that when they see the word '*share*' the question means division), they struggle because firstly, problems often don't appear to have 'key words' (or they can't identify them) and secondly, because even understanding key words doesn't always help in understanding the 'mathematics' involved in the situation.

This is where the **Close Reading** technique can be helpful as it helps pupils appreciate the full context of the problem rather than look for specific words whose meaning is context-dependent.

The importance of prior knowledge

The above examples demonstrate that when approaching mathematics, readers need to have significant prior knowledge of mathematical notation, vocabulary, and context. This can often appear to the untrained eye as a different language; there can be misinterpretations of terminology, symbols or notation that can leave the reader unable to understand the diagram/ mathematical problem.

This point cannot be understated and highlights some non-examples when thinking about implementation: disciplinary literacy in mathematics is not a bolt-on to the curriculum (which runs the risk of creating extra workload for the teachers) or an attempt to bring 'literacy' into mathematics through things like 'real-world' mathematics.

Mathematical disciplinary literacy is about supporting pupils to be literate in the unique ways of thinking and communicating in mathematics and **relies on a deep knowledge of mathematical concepts and ideas, the links between those concepts and ideas, and the explicit teaching of both of those points.**

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14. Shanahan, T & Shanahan, C. (2008). *Teaching Disciplinary Literacy to Adolescents: Rethinking Content Area Literacy*. Harvard Educational Review. 78.

Part 2
How-to guides for effective use of Disciplinary Literacy in Mathematics

Supporting pupils to understand mathematics

Before thinking about concrete ways in which we might support the development of pupils' disciplinary literacy, it's worth tying together some of the key points we've covered so far:

- **Mathematical disciplinary literacy is about supporting pupils to be literate in the unique ways of thinking and communicating in mathematics.**
- **Mathematics is communicated through the use of words, symbols, diagrams and equations.**
- **Mathematicians employ strategies such as Close Reading and Translation to help make sense of information.**
- **Becoming literate relies on a deep knowledge of mathematical concepts and ideas, for which there is no substitute.**

Given the unique challenges that mathematics can present, and alongside developing a deep knowledge of mathematics, why might a focus on disciplinary literacy help?

Research by Grimm suggests a link between reading achievement and mathematics achievement, with pupils who have higher early reading achievement also demonstrating greater conceptual understanding in, and application of, mathematics.[15] Fang suggests this can be a particular problem when it comes to problem-solving, specifically word problems.[16] As such, it appears that a key lever in improving pupils' mathematics outcomes is around improving their literacy: both generally (ability to read & write) and specifically (disciplinary literacy).

Thinking about how we might we do this, the EEF *Improving Literacy* guidance reports all refer to five strategies that the evidence suggests can support pupils with reading comprehension.[17] Due to the unique nature of mathematics it may not be appropriate to use them in the same way you would in a more text-based subject such as English or History, but we might adapt our approach by instead focusing more on:

- **Activate prior knowledge:** pupils think about what they already know about a topic from previous teaching, study or other experiences, such as self-study, and try to make meaningful links.
- **Make predictions:** pupils predict what is being asked of them in the problem as they read and process it.
- **Clarify uncertainty:** pupils identify areas of uncertainty, which may be individual words, symbols, diagrams etc, and seek information to clarify meaning.

15. Grimm K. J. (2008). *Longitudinal associations between reading and mathematics achievement*. Developmental neuropsychology, 33(3), 410–426
16. Fang, Z. (2023). *Demystifying Academic Reading*. Taylor & Francis.
17. EEF. *Reading Comprehension Strategies*. educationendowmentfoundation.org.uk/education-evidence/teaching-learning-toolkit/reading-comprehension-strategies

To support the effective utilisation of these strategies, it's critical to understand why they can be effective and what purpose each of them serve.

All three support the identification and use of relevant prior knowledge, with many studies showing the importance of prior knowledge (including the now popular quote from Ausubel, *'The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly'*[18]). However, a particular challenge in mathematics is selecting the correct prior knowledge when so many problems can seem superficially similar but relate to completely different parts of mathematics.

Consider the four mathematical terms below:

$32, \quad 3\frac{1}{2}, \quad 3y, \quad 3^2$

Each of these are superficially similar, they involve '3' followed by something.

In order to use them appropriately we need to not only activate prior knowledge, but the correct prior knowledge. To support the correct selection, we need to ensure that we have **clarity** over what is being asked, which might involve **making predictions** based on the context in which the question is being asked.

Let's consider the start of a question given below:

A helicopter leaves Bristol and flies due east for 10 miles. Then the helicopter flies 8 miles north before landing.

Even though a question hasn't yet been asked, an expert mathematician is likely to already be making predictions about what concepts the problem can relate to. They'll seek to improve their predictions by clarifying their understanding as they continue to read the scenario. The reflective questions in Table 1, below, can help with the use of these strategies.

Given all of the above, let's return to the central question we're trying to answer: how might we support our pupils in becoming mathematically literate in a classroom setting?

A starting point must be to recognise our own expertise. While expertise in our subject is critical in being able to communicate it effectively, it's a double-edged sword as it's also often the cause of our blind-spots or 'expert blindness'.[19] Our intimate familiarity with the material prevents us from seeing the challenges that novices (ie our pupils) are likely to face. Recognising this allows us to employ a key strategy in supporting pupils: thinking aloud.

Table 1

Possible questions	The (Adapted) 'Big 5' in use
1. **What is the key information given in the question?** 2. **What prior knowledge do you have already that might be helpful?** 3. **Have you seen a question like this before?**	Activate prior knowledge
1. **What might the question be asking us to do?** 2. **What mathematical concepts/ideas is the question likely to involve?** 3. **What information is useful/can be ignored?** 4. **What might I need to do in order to solve the problem?**	Make predictions
1. **Where do you find your attention drawn to?** 2. **In which order did you process the information in the question?** 3. **Can you justify the solution?** 4. **Can you put the information in the question into a more useful format eg a diagram/equation?**	Clarify uncertainty

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18. Ausubel, D. P. (2000). *The acquisition and retention of knowledge: A cognitive view*. Boston, MA: Kluwer.
19. Dr Carl Marci. *Expert Blindness: the silent threat to success.* warc.com/newsandopinion/opinion/expert-blindness-the-silent-threat-to-success/en-gb/3082

Using Think Aloud

Mulholland (2022) notes, Think Aloud strategies not only provide a model or worked example in which the process is explained, they also demonstrate *"how an 'expert' learner approaches a problem, making these invisible processes visible and accessible to pupils."* [20]

As previously mentioned, research suggests that pupil success rate can significantly increase when worded problems are represented symbolically – an experience we're sure will resonate with many mathematics teachers across phases and contexts (that sudden response from pupils of "oh that's what the question is asking" after we've represented it differently).

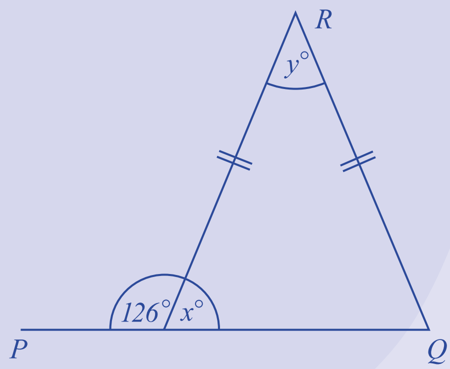
This suggests that, alongside explicit teaching of the underpinning mathematical knowledge, we can support pupils' mathematical literacy by also explicitly teaching how we make sense of problems. We can use the reflective questions shared previously to help pupils understand how we: activate prior knowledge, make predictions, and clarify uncertainty. Mulholland (2022) splits the strategy into three phases:

- **Plan:** This is where we look at a problem and describe how we are approaching it. What are the key bits of information we're attending to? What are we ignoring? What do we think the question is likely to be about? Why?
- **Monitor:** As we start working our way through the problem, we continually test the predictions made in the planning stage. Are they still valid? Or do we need to update them in light of new information?
- **Evaluate:** Finally, having completed the solution, we reflect on the process as a whole. Was our prediction correct? Can we check the answer?

Let's consider the question below in figure 3, and use the reflective questions in table 1, on page 11, to help us.

Figure 3[21]

*Diagram **NOT** accurately drawn*



PQ is a straight line

(a) Work out the size of the angle marked y^o

It's highly plausible that many mathematicians may not even get to the actual words – they might have started annotating the diagram to work out the missing angles, before reading the question and realising they've already solved it.

Using a 'Think Aloud' strategy as a teacher for this problem could involve talking pupils through that process explicitly, as exemplified in figure 4, on page 13.

20. EEF. *Thinking aloud to support mathematical problem solving*. https://educationendowmentfoundation.org.uk/news/eef-blog-thinking-aloud-to-support-mathematical-problem-solving

21. Adapted from Maths Genie https://www.mathsgenie.co.uk/resources/34_angles.pdf

Figure 4: Example of 'Think Aloud' strategy for geometry problem in figure 3 on page 12

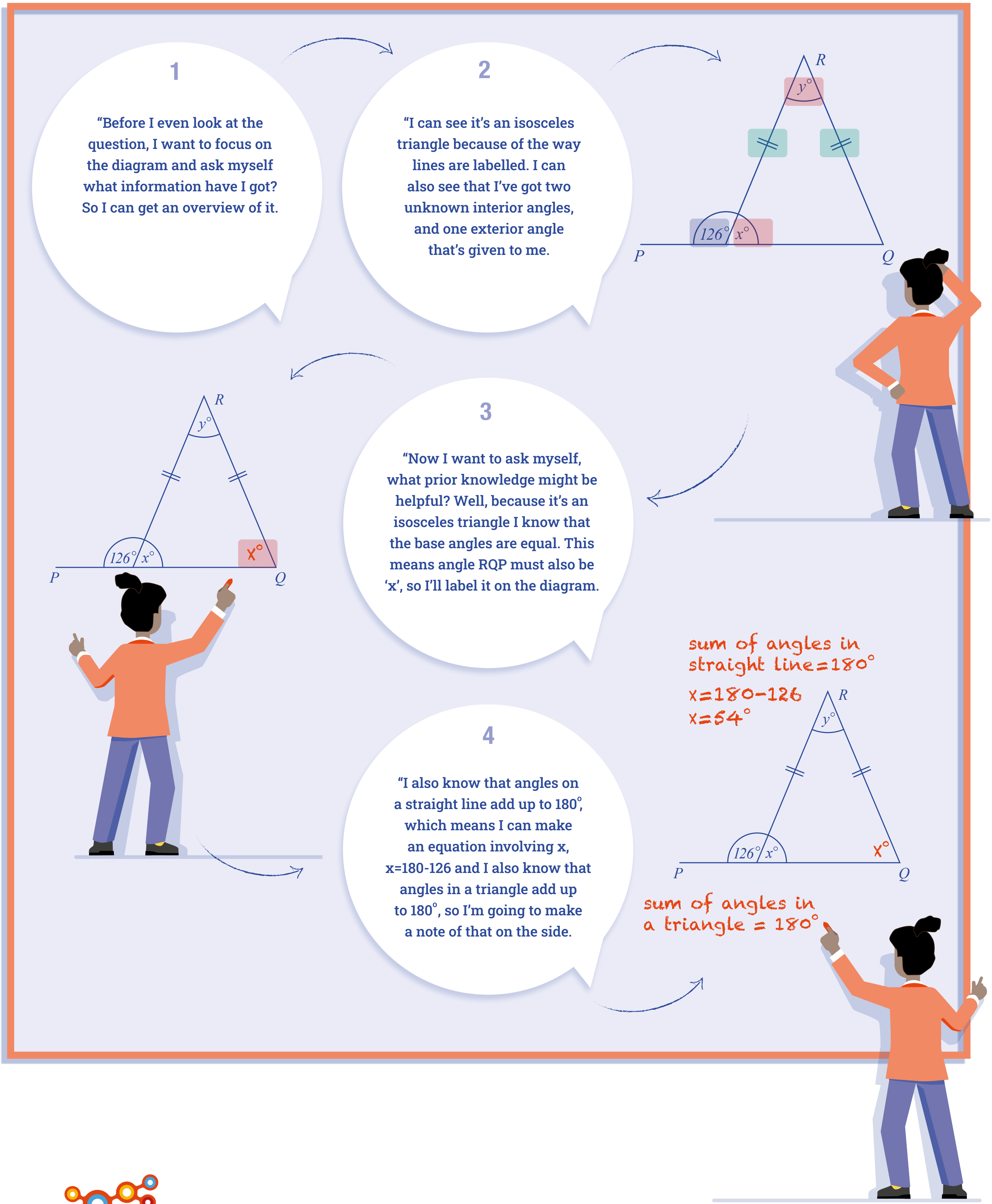


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Figure 4 continued: Example of 'Think Aloud' strategy for geometry problem in figure 3 on page 12

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"Now, I want to think about what information I might be able to add in based on what I've already identified because that's likely to help me get a better grasp of the situation.

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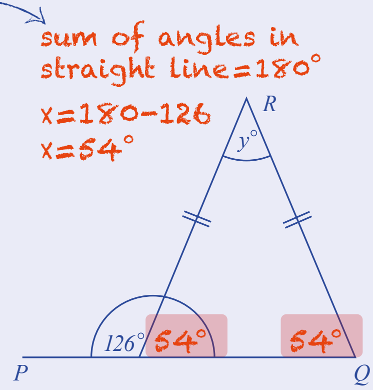
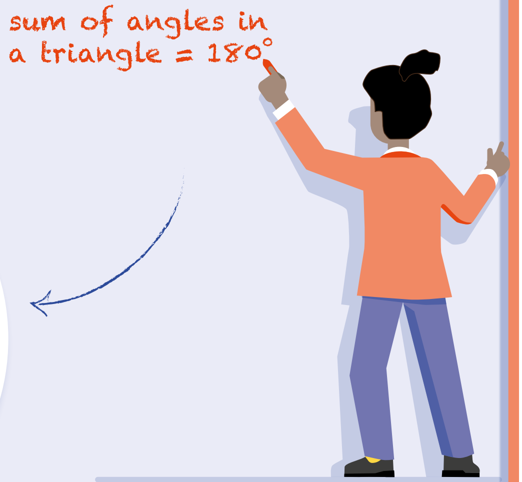
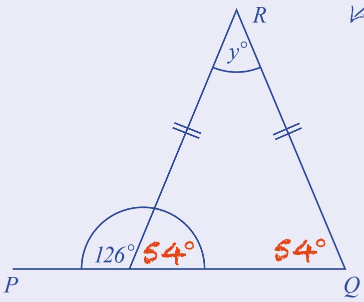
"I've already made an equation for x, and working that out gives me x=54°. This means I can now change my label for the other base angle to also being 54°.







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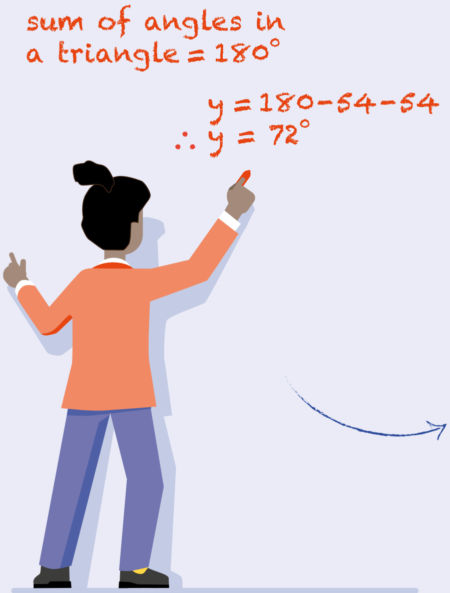
"Finally, it looks like me making that note of the sum of angles in a triangle has come in useful because I can now make an equation for my final missing angle, so y=180-54-54, therefore y=72°.



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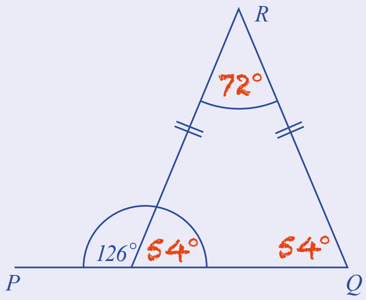
"Lastly, I'll re-read the question to make sure I've answered it and in this case, because of the prior knowledge we've drawn on to add information to the diagram, we've already found y so it looks like we're done."



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How this differs from a model or worked example is that we haven't talked pupils through a procedure, or used a question and talked through how we would answer that question. Instead, we've thought aloud about how we would process the information in the diagram, what we would do with it, and why.

The non-example of this could look like:

"I can work out x by doing 180-126, so x=54°.

"As it's an isosceles triangle I also know that angle RQP must be 54°.

"Finally, I can work out y by doing 180-54-54, so y=72°."

The mathematical content is exactly the same, but this non-example hasn't involved any explanation of how an expert mathematician might make sense of the information, it's overlooked "making these invisible processes visible and accessible to pupils".[22] Having done this, we can then rely on some of the metacognitive strategies from the EEF guidance report on *Improving Mathematics in Key Stages 2 and 3* to support pupils in monitoring and evaluating their progress or solution.[23]

Having considered how this could work in geometry, let's look at an example from algebra also by considering the problem in figure 5, below:

Figure 5

Kuldeep bought 3 pencils and a ruler.
It cost him £11.

Nelly bought 2 pencils and a ruler.
It cost her £8.

How much would 4 pencils cost?

Let's see how the 'Think Aloud' strategy would work for this problem in figure 6 on page 16.

It's worth mentioning here that these examples are meant to demonstrate how a Think Aloud strategy *could* work, not how they *should*. Classroom usage could involve much more questioning or be split up slightly differently, and the working and explanations might be done simultaneously rather than one followed by the other, being tailored to meet the needs of the individual classes and their unique prior knowledge.

22. EEF. *Thinking aloud to support mathematical problem solving*. https://educationendowmentfoundation.org.uk/news/eef-blog-thinking-aloud-to-support-mathematical-problem-solving

23. EEF. *Integrating Evidence into Maths Teaching. Planning a Think Aloud*. https://d2tic4wvo1iusb.cloudfront.net/production/eef-guidance-reports/maths-ks-2-3/Planning_a_Think_Aloud_1.0.pdf?v=1718753530

Figure 6: Example of 'Think Aloud' strategy for algebra problem in figure 5 on page 15

1

"It looks like there are some sums going on here, and I'm being asked to work out the total cost of something, but because there's a lot of information here and it's not written in a very helpful format I'm not 100% sure about that just yet. So the first thing I want to do is translate this into a format that's more useful to me.

2

"Both of these people have bought a sum of items, so maybe I can represent this as an equation? If I let 'p' be the number of pencils, and 'r' be the number of rulers, that means for Kuldeep, 3p and r must add up to £11. So I could write:

$3p + r = 11$

3

"So I've got an equation for Kuldeep, can I do something similar for Nelly? Yes, I think I can. For Nelly I could write:

$2p + r = 8$

4

"What's different about these equations to the ones we normally see? Yes, these equations have 2 unknowns. But because they're related we can solve them simultaneously. So we have:

$3p + r = 11$
$2p + r = 8$

5

"It looks like the only difference between the unknowns of the equations is 1p, and the total is 3 (because 11-8=3), so it looks like 1 pencil costs £3. But I want to solve this mathematically to make sure I'm right.

6

"What I want to do in this case is subtract one equation from the other, just like we would for individual terms. That's why I've lined up my equations one above the other. So 3p subtract 2p would give me just p; r subtract r would leave me with nothing; and 11 subtract 8 would leave me with 3. So I'd get this:

$p = 3$

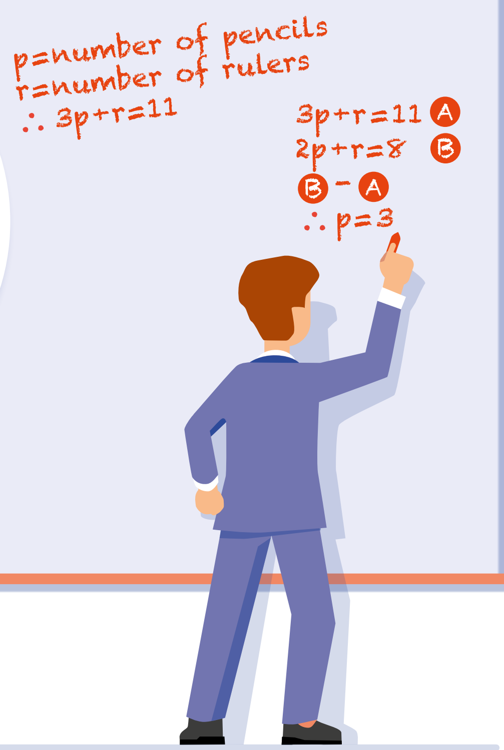
7

"So it looks like we were right! This supports our earlier prediction, but we're not finished yet. The question asks me to calculate the total for 4 pencils, so I'd need to multiply my answer by 4:

$4p = 12$

8

And remembering to include my units will give me a final answer of £12."



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An approach to worded problems

As discussed previously, worded problems present an additional challenge in mathematics. Not only do pupils need to be able to execute the relevant mathematical procedures, they also have to make sense of a problem that is often presented using words but in a way unlike other subjects.

This can be a challenge, but most mathematical word problems (at least at school level) are laid out in relatively similar ways. We can support pupils with making sense of these problems by helping them to recognise some of the common features between how mathematics problems are laid out, as the example below, in figure 7, from Feeney demonstrates.[24]

Although we've already mentioned this earlier, because this point is so critical we feel it's worth including here again: **no 'strategy' will be an adequate substitute for a lack of underlying knowledge and proficiency**. Instead, these strategies are best thought about as being **ways to streamline attention** and support pupils to make appropriate and relevant links to their existing knowledge.

By supporting pupils to recognise the patterns involved, we can help them answer some of the reflective questions posed earlier (eg what information is useful/can be ignored? What might the question be asking us to do?), and separate out the processes that worded problems often require pupils to engage in: understanding the question, translating it into a more meaningful format, executing a procedure, finding a solution.

By narrowing their focus onto one of those steps at a time and reducing irrelevant, distracting information, it's likely that we'll be supporting pupils' limited processing and, in doing so, also making it more likely that they will succeed.

Figure 7

Amal drives her car for work.

She claims 40p per mile from her employer.

Amal's car travels 52 miles for each gallon of petrol.

She pays £5.36 per gallon for petrol. On one journey Amal drives 260 miles

For this journey, how much more does she claim than she pays for petrol?

Answer £______________

Scenario
Understand the context

Information
Identify the information you need

Question
Select the Maths you need to solve the question.

24. Feeney, C. (2021). *Read like a Mathematician*. *Language Conscious T & L.* https://clarefeeneyuk.com/?p=69